General Certificate of Education

Mathematics 6360

MFP1 Further Pure 1

Mark Scheme

2010 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Dr Michael Cresswell Director General

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comments
1(a)	$\alpha+\beta=2,\ \alpha\beta=\frac{1}{3}$	B1B1	2	
(b)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1		or other appropriate formula
	$\ldots = 8-3(\frac{1}{3})(2)=6$	m1A1	3	m1 for substn of numerical values; A1 for result shown (AG)
(c)	Sum of roots = $\dfrac{\alpha^3+\beta^3}{\alpha\beta}$	M1		
	$\ldots=\dfrac{6}{\frac{1}{3}}=18$	A1F		ft wrong value for $\alpha\beta$
	Product = $\alpha\beta=\frac{1}{3}$	B1F		ditto
	Equation is $3x^2-54x+1=0$	A1F	4	Integer coeffs and "= 0" needed; ft wrong sum and/or product
	Total		**9**	
2(a)	$z^2=1+2i+i^2=2i$	M1A1	2	M1 for use of $i^2=-1$
(b)	$z^8=(2i)^4$	M1		or equivalent complete method
	$\ldots=16i^4=16$	A1	2	convincingly shown (AG)
(c)	$(z^*)^2=(1-i)^2$	M1		for use of $z^*=1-i$
	$\ldots=-2i=-z^2$	A1	2	convincingly shown (AG)
	Total		**6**	
3	$\sin\frac{\pi}{2}=1$ stated or used	B1		Deg/dec penalised in 4th mark
	Introduction of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $4x+\frac{\pi}{4}$ to x	m1		incl division of all terms by 4
	$x=\dfrac{\pi}{16}+\dfrac{1}{2}n\pi$	A1	4	or equivalent unsimplified form
	Total		**4**	
4(a)	$\mathbf{I}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$	B1		stated or used at any stage
	Attempt at $(\mathbf{A}-\mathbf{I})^2$	M1		with at most one numerical error
	$(\mathbf{A}-\mathbf{I})^2=\begin{bmatrix}0&4\\3&0\end{bmatrix}\begin{bmatrix}0&4\\3&0\end{bmatrix}=12\mathbf{I}$	A1	3	
(b)	$\mathbf{A}-\mathbf{B}=\begin{bmatrix}0&1\\3-p&0\end{bmatrix}$	B1		
	$(\mathbf{A}-\mathbf{B})^2=\begin{bmatrix}3-p&0\\0&3-p\end{bmatrix}$	M1A1		M1 A0 if 3 entries correct
	$\ldots=(\mathbf{A}-\mathbf{I})^2$ for $p=-9$	A1F	4	ft wrong value of k
	Total		**7**	

MFP1

Q	Solution	Mark	Total	Comments
5(a)	$x^{-\frac{1}{2}} \to \infty$ as $x \to 0$	E1	1	Condone "$x^{-\frac{1}{2}}$ has no value at $x = 0$"
(b)(i)	$\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}}\ (+c)$	M1A1		M1 for correct power of x
	$\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,dx = \frac{1}{2}$	A1F	3	ft wrong coefficient of $x^{\frac{1}{2}}$
(ii)	$\int x^{-\frac{5}{4}}\,dx = -4x^{-\frac{1}{4}}\ (+c)$	M1A1		M1 for correct power of x
	$x^{-\frac{1}{4}} \to \infty$ as $x \to 0$, so no value	E1F	3	ft wrong coefficient of $x^{-\frac{1}{4}}$
	Total		**7**	
6(a)(i)	Coords (3, 2), (9, 2), (9, 4), (3, 4)	M1A1	2	M1 for multn of x by 3 or y by 2 (PI)
(ii)	R_2 shown correctly on insert	B1	1	
(b)(i)	R_3 shown correctly on insert	B2,1F	2	B1 for rectangle with 2 vertices correct; ft if c's R_2 is a rectangle in 1st quad
(ii)	Matrix of rotation is $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(c)	Multiplication of matrices	M1		(either way) or other complete method
	Required matrix is $\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$	A1	2	
	Total		**8**	
7(a)(i)	Asymptotes $x = 2$, $y = 0$	B1B1	2	
(ii)	One correct branch	B1		
	Both branches correct	B1	2	no extra branches; $x = 2$ shown
(b)(i)	$f(3) = -1$, $f(4) = 3$	B1		where $f(x) = (x-3)(x-2)^2 - 1$; OE
	Sign change, so α between 3 and 4	E1	2	
(ii)	$f(3.5)$ considered first	M1		OE but must consider $x = 3.5$
	$f(3.5) > 0$ so $3 < \alpha < 3.5$	A1		Some numerical value(s) needed
	$f(3.25) < 0$ so $3.25 < \alpha < 3.5$	A1	3	Condone absence of values here
	Total		**9**	

MFP1

Q	Solution	Mark	Total	Comments
8(a)	$\Sigma r^3 + \Sigma r = \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)$	M1		at least one term correct
	Factor n clearly shown	m1		or $n + 1$ clearly shown to be a factor
	$\ldots = \frac{1}{4}n(n+1)(n^2+n+2)$	A1A1	4	OE; A1 for $\frac{1}{4}$, A1 for quadratic
(b)	Valid equation formed	M1		
	Factors n, $n + 1$ removed	m1		
	$3n^2 - 29n - 10 = 0$	A1		OE
	Valid factorisation or solution	m1		of the correct quadratic
	$n = 10$ is the only pos int solution	A1	5	SC 1/2 for $n = 10$ after correct quad
	Total		**9**	
9(a)	$x = 2, y = 0 \Rightarrow \frac{4}{a^2} - 0 = 1$ so $a = 2$	E2,1		E1 for verif'n or incomplete proof
	Asymps $\Rightarrow \pm\frac{b}{a} = \pm 2$ so $b = 2a = 4$	E2,1	4	ditto
(b)	Line is $y - 0 = m(x - 1)$	B1		OE
	Elimination of y	M1		
	$4x^2 - m^2(x^2 - 2x + 1) = 16$	A1		OE (no fractions)
	So $(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0$	A1	4	convincingly shown (AG)
(c)	Discriminant equated to zero	M1		
	$4m^4 - 4m^4 - 64m^2 + 16m^2 + 256 = 0$	A1		OE
	$-3m^2 + 16 = 0$, hence result	A1	3	convincingly shown (AG)
(d)	$m^2 = \frac{16}{3} \Rightarrow \frac{4}{3}x^2 - \frac{32}{3}x + \frac{64}{3} = 0$	M1		
	$x^2 - 8x + 16 = 0$, so $x = 4$	m1A1		
	Method for y-coordinates	m1		using $m = \pm\frac{4}{\sqrt{3}}$ or from equation of hyperbola; dep't on previous m1
	$y = \pm 4\sqrt{3}$	A1	5	
	Total		**16**	
	TOTAL		**75**	